SL 1.1

Content	Guidance, clarification and syllabus links
Operations with numbers in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer.	Calculator or computer notation is **not** acceptable. For example, 5.2E30 is **not** acceptable and should be written as 5.2×10^{30}.

SL 1.2

Content	Guidance, clarification and syllabus links
Arithmetic sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways. If technology is used in examinations, students will be expected to identify the first term and the common difference.
Applications.	Examples include simple interest over a number of years.
Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.	Students will need to approximate common differences.

SL 1.3

Content	Guidance, clarification and syllabus links
Geometric sequences and series. Use of the formulae for the nth term and the sum of the first n terms of the sequence.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways.
Use of sigma notation for the sums of geometric sequences.	If technology is used in examinations, students will be expected to identify the first term and the ratio. **Link to:** models/functions in topic 2 and regression in topic 4.
Applications.	Examples include the spread of disease, salary increase and decrease and population growth.

SL 1.4

Content	Guidance, clarification and syllabus links
Financial applications of geometric sequences and series: • compound interest • annual depreciation.	Examination questions may require the use of technology, including built-in financial packages. The concept of simple interest may be used as an introduction to compound interest. Calculate the real value of an investment with an interest rate and an inflation rate. In examinations, questions that ask students to derive the formula will not be set. Compound interest can be calculated yearly, half-yearly, quarterly or monthly. **Link to**: exponential models/functions in topic 2.

SL 1.5

Content	Guidance, clarification and syllabus links
Laws of exponents with integer exponents.	**Examples:** $5^3 \times 5^{-6} = 5^{-3}, 6^4 \div 6^3 = 6, (2^3)^{-4} = 2^{-12},$ $(2x)^4 = 16x^4, 2x^{-3} = \frac{2}{x^3}$
Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.	Awareness that $a^x = b$ is equivalent to $\log_a b = x$, that $b > 0$, and $\log_e x = \ln x$.

SL 1.6

Content	Guidance, clarification and syllabus links
Approximation: decimal places, significant figures.	Students should be able to choose an appropriate degree of accuracy based on given data.
Upper and lower bounds of rounded numbers.	If $x = 4.1$ to one decimal place, $4.05 \leq x < 4.15$.
Percentage errors.	Students should be aware of, and able to calculate, measurement errors (such as rounding errors or measurement limitations). For example finding the maximum percentage error in the area of a circle if the radius measured is 2.5 cm to one decimal place.
Estimation.	Students should be able to recognize whether the results of calculations are reasonable. For example lengths cannot be negative.

SL 1.7

Content	Guidance, clarification and syllabus links
Amortization and annuities using technology.	Technology includes the built-in financial packages of graphic display calculators, spreadsheets. In examinations the payments will be made at the end of the period. Knowledge of the annuity formula will enhance understanding but will not be examined. **Link to**: exponential models (SL 2.5).

SL 1.8

Content	Guidance, clarification and syllabus links
Use technology to solve: • Systems of linear equations in up to 3 variables • Polynomial equations	In examinations, no specific method of solution will be required. In examinations, there will always be a unique solution to a system of equations. Standard terminology, such as zeros or roots, should be taught. **Link to:** quadratic models (SL 2.5)

AHL 1.9

Content	Guidance, clarification and syllabus links
Laws of logarithms: $\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m\log_a x$ for $a,\ x,\ y > 0$	In examinations, a will equal 10 or e. **Link to:** scaling large and small numbers (AHL 2.10).

AHL 1.10

Content	Guidance, clarification and syllabus links
Simplifying expressions, both numerically and algebraically, involving rational exponents.	**Examples:** $5^{\frac{1}{2}} \times 5^{\frac{1}{3}} = 5^{\frac{5}{6}}$, $6^{\frac{3}{4}} \div 6^{\frac{1}{2}} = 6^{\frac{1}{4}}$, $32^{\frac{3}{5}} = 8$, $x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

AHL 1.11

Content	Guidance, clarification and syllabus links
The sum of infinite geometric sequences.	**Link to:** the concept of a limit (SL 5.1), fractals (AHL 3.9), and Markov chains (AHL 4.19).

AHL 1.12

Content	Guidance, clarification and syllabus links
Complex numbers: the number i such that $i^2 = -1$. Cartesian form: $z = a + bi$; the terms real part, imaginary part, conjugate, modulus and argument. Calculate sums, differences, products, quotients, by hand and with technology. Calculating powers of complex numbers, in Cartesian form, with technology.	
The complex plane. Complex numbers as solutions to quadratic equations of the form $ax^2 + bx + c = 0$, $a \neq 0$, with real coefficients where $b^2 - 4ac < 0$.	Use and draw Argand diagrams. Quadratic formula and the link with the graph of $f(x) = ax^2 + bx + c$.

AHL 1.13

Content	Guidance, clarification and syllabus links
Modulus–argument (polar) form: $z = r(\cos\theta + i\sin\theta) = r\mathrm{cis}\theta$.	
Exponential form: $z = re^{i\theta}$.	Exponential form is sometimes called the Euler form.
Conversion between Cartesian, polar and exponential forms, by hand and with technology.	
Calculate products, quotients and integer powers in polar or exponential forms.	In examinations students will not be required to find the roots of complex numbers.
Adding sinusoidal functions with the same frequencies but different phase shift angles.	Phase shift and voltage in circuits as complex quantities. **Example:** Two AC voltages sources are connected in a circuit. If $V_1 = 10\cos(40t)$ and $V_2 = 20\cos(40t + 10)$ find an expression for the total voltage in the form $V = A\cos(40t + B)$.
Geometric interpretation of complex numbers.	Addition and subtraction of complex numbers can be represented as vector addition and subtraction. Multiplication of complex numbers can be represented as a rotation and a stretch in the Argand diagram.

AHL 1.14

Content	Guidance, clarification and syllabus links
Definition of a matrix: the terms element, row, column and order for $m \times n$ matrices.	
Algebra of matrices: equality; addition; subtraction; multiplication by a scalar for $m \times n$ matrices.	Including use of technology.
Multiplication of matrices. Properties of matrix multiplication: associativity, distributivity and non-commutativity.	Multiplying matrices to solve practical problems.
Identity and zero matrices. Determinants and inverses of $n \times n$ matrices with technology, and by hand for 2×2 matrices.	Students should be familiar with the notation $\boldsymbol{I}$ and $\boldsymbol{0}$.
Awareness that a system of linear equations can be written in the form $\boldsymbol{A}x = b$.	In examinations $\boldsymbol{A}$ will always be an invertible matrix, except when solving for eigenvectors.
Solution of the systems of equations using inverse matrix.	Model and solve real-life problems including: Coding and decoding messages Solving systems of equations. **Link to:** Markov chains (AHL 4.19), transition matrices (AHL 4.19) and phase portrait (AHL 5.17).

AHL 1.15

Content	Guidance, clarification and syllabus links
Eigenvalues and eigenvectors. Characteristic polynomial of 2×2 matrices. Diagonalization of 2×2 matrices (restricted to the case where there are distinct real eigenvalues).	Students will only be expected to perform calculations by hand and with technology for 2×2 matrices.
Applications to powers of 2×2 matrices.	Applications, for example movement of population between two towns, predator/prey models. $\boldsymbol{M}^n = \boldsymbol{P}\boldsymbol{D}^n\boldsymbol{P}^{-1}$, where $\boldsymbol{P}$ is a matrix of eigenvectors, and $\boldsymbol{D}$ is a diagonal matrix of eigenvalues. **Link to:** coupled differential equations (AHL 5.17).

SL 2.1

Content	Guidance, clarification and syllabus links
Different forms of the equation of a straight line. Gradient; intercepts. Lines with gradients m_1 and m_2 Parallel lines $m_1 = m_2$. Perpendicular lines $m_1 \times m_2 = -1$.	$y = mx + c$ (gradient-intercept form). $ax + by + d = 0$ (general form). $y - y_1 = m(x - x_1)$ (point-gradient form). Calculate gradients of inclines such as mountain roads, bridges, etc.

SL 2.2

Content	Guidance, clarification and syllabus links
Concept of a function, domain, range and graph. Function notation, for example $f(x)$, $v(t)$, $C(n)$. The concept of a function as a mathematical model.	**Example:** $f(x) = \sqrt{2-x}$, the domain is $x \leq 2$, range is $f(x) \geq 0$. A graph is helpful in visualizing the range.
Informal concept that an inverse function reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$.	**Example:** Solving $f(x) = 10$ is equivalent to finding $f^{-1}(10)$. Students should be aware that inverse functions exist for one to one functions; the domain of $f^{-1}(x)$ is equal to the range of $f(x)$.

SL 2.3

Content	Guidance, clarification and syllabus links
The graph of a function; its equation $y = f(x)$.	Students should be aware of the difference between the command terms "draw" and "sketch".
Creating a sketch from information given or a context, including transferring a graph from screen to paper. Using technology to graph functions including their sums and differences.	All axes and key features should be labelled. This may include functions not specifically mentioned in topic 2.

SL 2.4

Content	Guidance, clarification and syllabus links
Determine key features of graphs.	Maximum and minimum values; intercepts; symmetry; vertex; zeros of functions or roots of equations; vertical and horizontal asymptotes using graphing technology.
Finding the point of intersection of two curves or lines using technology.	

SL 2.5

Content	Guidance, clarification and syllabus links
Modelling with the following functions:	
Linear models. $f(x) = mx + c$.	Including piecewise linear models, for example horizontal distances of an object to a wall, depth of a swimming pool, mobile phone charges. **Link to:** equation of a straight line (SL 2.1) and arithmetic sequences (SL 1.2).
Quadratic models. $f(x) = ax^2 + bx + c \quad ; \quad a \neq 0$. Axis of symmetry, vertex, zeros and roots, intercepts on the x-axis and y-axis.	Technology can be used to find roots. **Link to:** use of technology to solve quadratic equations (SL 1.8).
Exponential growth and decay models. $f(x) = ka^x + c$ $f(x) = ka^{-x} + c \quad (\text{for } a > 0)$ $f(x) = ke^{rx} + c$ Equation of a horizontal asymptote.	**Link to:** compound interest (SL 1.4), geometric sequences and series (SL 1.3) and amortization (SL 1.7).
Direct/inverse variation: $f(x) = ax^n, \quad n \in \mathbb{Z}$ The y-axis as a vertical asymptote when $n < 0$.	
Cubic models: $f(x) = ax^3 + bx^2 + cx + d$.	
Sinusoidal models: $f(x) = a\sin(bx) + d, \quad f(x) = a\cos(bx) + d$.	Students will **not** be expected to translate between $\sin x$ and $\cos x$, and will only be required to predict or find amplitude (a), period ($\frac{360^\circ}{b}$), or equation of the principal axis ($y = d$).

SL 2.6

Content	Guidance, clarification and syllabus links
Modelling skills: Use the modelling process described in the "mathematical modelling" section to create, fit and use the theoretical models in section SL2.5 and their graphs.	Fitting models using regression is covered in topic 4. **Link to**: theoretical models (SL 2.5) to be used to develop the modelling skills and, for HL students, (AHL 2.9).
Develop and fit the model: Given a context recognize and choose an appropriate model and possible parameters. Determine a reasonable domain for a model.	
Find the parameters of a model.	By setting up and solving equations simultaneously (using technology), by consideration of initial conditions or by substitution of points into a given function. At SL, students will not be expected to perform non-linear regressions, but will be expected to set up and solve up to three linear equations in three variables using technology.
Test and reflect upon the model: Comment on the appropriateness and reasonableness of a model. Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation.	
Use the model: Reading, interpreting and making predictions based on the model.	Students should be aware of the dangers of extrapolation.

AHL 2.7

Content	Guidance, clarification and syllabus links
Composite functions in context. The notation $(f \circ g)(x) = f(g(x))$. Inverse function f^{-1}, including domain restriction. Finding an inverse function.	$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$. **Example:** $f(x) = (x-3)^2 - 2$ has an inverse if the domain is restricted to $x \geq 3$ or to $x \leq 3$.

AHL 2.8

Content	Guidance, clarification and syllabus links
Transformations of graphs.	Students will be expected to be able to perform transformations on all functions from the SL and AHL section of this topic, and others in the context of modelling real-life situations.
Translations: $y = f(x) + b$; $y = f(x-a)$. Reflections: in the x axis $y = -f(x)$, and in the y axis $y = f(-x)$.	Translation by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ denotes horizontal translation of 3 units to the right, and vertical translation of 2 units down.
Vertical stretch with scale factor p: $y = pf(x)$. Horizontal stretch with scale factor $\frac{1}{q}$: $y = f(qx)$	x and y axes are invariant.
Composite transformations.	Students should be made aware of the significance of the order of transformations. **Example:** $y = x^2$ used to obtain $y = 3x^2 + 2$ by a vertical stretch of scale factor 3 followed by a translation of $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. **Example:** $y = \sin x$ used to obtain $y = 4\sin 2x$ by a vertical stretch of scale factor 4 and a horizontal stretch of scale factor $\frac{1}{2}$.

AHL 2.9

Content	Guidance, clarification and syllabus links
In addition to the models covered in the SL content the AHL content extends this to include modelling with the following functions: Exponential models to calculate half-life.	**Link to:** modelling skills (SL2.6).
Natural logarithmic models: $f(x) = a + b\ln x$	
Sinusoidal models: $f(x) = a\sin(b(x - c)) + d$	Radian measure should be assumed unless otherwise indicated by the use of the degree symbol, for example with $f(x) = \sin x°$. In radians, period is $\frac{2\pi}{b}$. Students should be aware that a horizontal translation of c can be referred to as a phase shift. **Link to:** radian measure (AHL 3.7)
Logistic models: $f(x) = \frac{L}{1 + Ce^{-kx}}; L, \quad C, \quad k > 0$	The logistic function is used in situations where there is a restriction on the growth. For example population on an island, bacteria in a petri dish or the increase in height of a person or seedling. Horizontal asymptote at $f(x) = L$ is often referred to as the carrying capacity.
Piecewise models.	In some cases, parameters may need to be found that ensure continuity of the function, for example find a to make $f(x) = \begin{cases} 1 + x, & 0 \leq x < 2 \\ ax^2 + x, & x \geq 2 \end{cases}$ continuous. The formal definition of continuity is not required. In examinations, students may be expected to interpret and use other models that are introduced in the question.

AHL 2.10

Content	Guidance, clarification and syllabus links
Scaling very large or small numbers using logarithms. Linearizing data using logarithms to determine if the data has an exponential or a power relationship using best-fit straight lines to determine parameters.	Choosing a manageable scale, for example for data with a wide range of values in one, or both variables and/or where the emphasis of a graph is the rate of growth, rather than the absolute value. **Link to:** laws of logarithms (AHL 1.9) and Pearson's product moment correlation coefficient (SL 4.4).
Interpretation of log-log and semi-log graphs.	In examinations, students will not be expected to draw or sketch these graphs.

SL 3.1

Content	Guidance, clarification and syllabus links
The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.	In SL examinations, only right-angled trigonometry questions will be set in reference to three-dimensional shapes. In problems related to these topics, students should be able to identify relevant right-angled triangles in three-dimensional objects and use them to find unknown lengths and angles.

SL 3.2

Content	Guidance, clarification and syllabus links
Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles.	In all areas of this topic, students should be encouraged to sketch well-labelled diagrams to support their solutions. **Link to:** inverse functions (SL2.2) when finding angles.
The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ Area of a triangle as $\frac{1}{2}ab\sin C$.	This section does not include the ambiguous case of the sine rule.

SL 3.3

Content	Guidance, clarification and syllabus links
Applications of right and non-right angled trigonometry, including Pythagoras' theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.	Contexts may include use of bearings.

SL 3.4

Content	Guidance, clarification and syllabus links
The circle: length of an arc; area of a sector.	Radians are not required at SL.

SL 3.5

Content	Guidance, clarification and syllabus links
Equations of perpendicular bisectors.	Given either two points, or the equation of a line segment and its midpoint. **Link to:** equations of straight lines (SL 2.1).

SL 3.6

Content	Guidance, clarification and syllabus links
Voronoi diagrams: sites, vertices, edges, cells. Addition of a site to an existing Voronoi diagram. Nearest neighbour interpolation. Applications of the "toxic waste dump" problem.	In examinations, coordinates of sites for calculating the perpendicular bisector equations will be given. Students will not be required to construct perpendicular bisectors. Questions may include finding the equation of a boundary, identifying the site closest to a given point, or calculating the area of a region. All points within a cell can be estimated to have the same value (e.g. rainfall) as the value of the site. In examinations, the solution point will always be at an intersection of three edges. **Contexts:** Urban planning, spread of diseases, ecology, meteorology, resource management.

AHL 3.7

Content	Guidance, clarification and syllabus links
The definition of a radian and conversion between degrees and radians. Using radians to calculate area of sector, length of arc.	Radian measure may be expressed as exact multiples of π, or decimals. **Link to**: trigonometric functions (AHL 2.9).

AHL 3.8

Content	Guidance, clarification and syllabus links
The definitions of $\cos\theta$ and $\sin\theta$ in terms of the unit circle. The Pythagorean identity: $\cos^2\theta + \sin^2\theta = 1$ Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$ Extension of the sine rule to the ambiguous case.	Students should understand how the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ can be constructed from the unit circle. Knowledge of exact values of $\cos\theta$, $\sin\theta$, and $\tan\theta$ will not be assessed on examinations, but may aid student understanding of trigonometric functions.
Graphical methods of solving trigonometric equations in a finite interval.	**Link to:** sinusoidal models (SL2.5 and AHL2.9).

AHL 3.9

Content	Guidance, clarification and syllabus links
Geometric transformations of points in two dimensions using matrices: reflections, horizontal and vertical stretches, enlargements, translations and rotations.	Matrix transformations of the form: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix}$. **Link to:** matrices (AHL 1.14).
Compositions of the above transformations.	Iterative techniques to generate fractals. **Link to:** infinite geometric series (AHL 1.11) and Markov chains (AHL 4.19).
Geometric interpretation of the determinant of a transformation matrix.	Area of image $= \lvert \det \boldsymbol{A} \rvert \times$ area of object.

AHL 3.10

Content	Guidance, clarification and syllabus links
Concept of a vector and a scalar. Representation of vectors using directed line segments. Unit vectors; base vectors i, j, k. Components of a vector; column representation; $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$ The zero vector $\boldsymbol{0}$, the vector $-\boldsymbol{v}$.	Use algebraic and geometric approaches to calculate the sum and difference of two vectors, multiplication by a scalar, kv (parallel vectors), magnitude of a vector $\lvert \boldsymbol{v} \rvert$ from components. The resultant as the sum of two or more vectors.
Position vectors $\overrightarrow{OA} = a$.	
Rescaling and normalizing vectors.	$\frac{\boldsymbol{v}}{\lvert \boldsymbol{v} \rvert}$, the unit normal vector. **Example:** Find the velocity of a particle with speed 7ms^{-1} in the direction $3i + 4j$.

AHL 3.11

Content	Guidance, clarification and syllabus links
Vector equation of a line in two and three dimensions: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$, where $\boldsymbol{b}$ is a direction vector of the line.	Convert to parametric form: $x = x_0 + \lambda l, y = y_0 + \lambda m, z = z_0 + \lambda n$.

AHL 3.12

Content	Guidance, clarification and syllabus links
Vector applications to kinematics. Modelling linear motion with constant velocity in two and three dimensions.	Finding positions, intersections, describing paths, finding times and distances when two objects are closest to each other. $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$. Relative position of B from A is $\overrightarrow{AB}$.
Motion with variable velocity in two dimensions.	**For example:** $\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 7 \\ 6-4t \end{pmatrix}$. Projectile motion and circular motion are special cases. $f(t-a)$ to indicate a time-shift of a. **Link to:** kinematics (AHL 5.13) and phase shift (AHL 1.13).

AHL 3.13

Content	Guidance, clarification and syllabus links
Definition and calculation of the scalar product of two vectors. The angle between two vectors; the acute angle between two lines.	Calculate the angle between two vectors using $\boldsymbol{v} \cdot \boldsymbol{w} = \|\boldsymbol{v}\|\|\boldsymbol{w}\|\cos\theta$, where θ is the angle between two non-zero vectors $\boldsymbol{v}$ and w, and ascertain whether the vectors are perpendicular ($\boldsymbol{v} \cdot \boldsymbol{w} = 0$).
Definition and calculation of the vector product of two vectors.	$\boldsymbol{v} \times \boldsymbol{w} = \|\boldsymbol{v}\|\|\boldsymbol{w}\|\sin\theta\boldsymbol{n}$, where θ is the angle between v and $\boldsymbol{w}$ and $\boldsymbol{n}$ is the unit normal vector whose direction is given by the right-hand screw rule. **Not required:** generalized properties and proofs of scalar and cross product.
Geometric interpretation of $\|\boldsymbol{v} \times \boldsymbol{w}\|$.	Use of $\|\boldsymbol{v} \times \boldsymbol{w}\|$ to find the area of a parallelogram (and hence a triangle).
Components of vectors.	The component of vector a acting in the direction of vector $\boldsymbol{b}$ is $\frac{\boldsymbol{a} \cdot \boldsymbol{b}}{\|\boldsymbol{b}\|} = \|\boldsymbol{a}\|\cos\theta$. The component of a vector a acting perpendicular to vector b, in the plane formed by the two vectors, is $\frac{\|\boldsymbol{a} \times \boldsymbol{b}\|}{\|\boldsymbol{b}\|} = \|\boldsymbol{a}\|\sin\theta$.

AHL 3.14

Content	Guidance, clarification and syllabus links
Graph theory: Graphs, vertices, edges, adjacent vertices, adjacent edges. Degree of a vertex.	Students should be able to represent real-world structures (circuits, maps, etc) as graphs (weighted and unweighted).
Simple graphs; complete graphs; weighted graphs.	Knowledge of the terms connected and strongly connected.
Directed graphs; in degree and out degree of a directed graph. Subgraphs; trees.	**Link to**: matrices (AHL 1.14).

AHL 3.15

Content	Guidance, clarification and syllabus links
Adjacency matrices. Walks. Number of k -length walks (or less than k -length walks) between two vertices.	Given an adjacency matrix A, the $(i, \quad j)^{\text{th}}$ entry of A^k gives the number of k length walks connecting i and j.
Weighted adjacency tables. Construction of the transition matrix for a strongly-connected, undirected or directed graph.	Weights could be costs, distances, lengths of time for example. Consideration of simple graphs, including the Google PageRank algorithm as an example of this. **Link to:** transition matrices and Markov chains (AHL 4.19).

AHL 3.16

Content	Guidance, clarification and syllabus links
Tree and cycle algorithms with undirected graphs. Walks, trails, paths, circuits, cycles.	
Eulerian trails and circuits. Hamiltonian paths and cycles. Minimum spanning tree (MST) graph algorithms: Kruskal's and Prim's algorithms for finding minimum spanning trees.	Determine whether an Eulerian trail or circuit exists. Use of matrix method for Prim's algorithm.
Chinese postman problem and algorithm for solution, to determine the shortest route around a weighted graph with up to four odd vertices, going along each edge at least once.	Students should be able to explain why the algorithm for constructing the Chinese postman problem works, apply the algorithm and justify their choice of algorithm.
Travelling salesman problem to determine the Hamiltonian cycle of least weight in a weighted complete graph. Nearest neighbour algorithm for determining an upper bound for the travelling salesman problem. Deleted vertex algorithm for determining a lower bound for the travelling salesman problem.	Practical problems should be converted to the classical problem by completion of a table of least distances where necessary.

SL 4.1

Content	Guidance, clarification and syllabus links
Concepts of population, sample, random sample, discrete and continuous data.	This is designed to cover the key questions that students should ask when they see a data set/ analysis.
Reliability of data sources and bias in sampling.	Dealing with missing data, errors in the recording of data.
Interpretation of outliers.	Outlier is defined as a data item which is more than $1.5 \times$ interquartile range (IQR) from the nearest quartile. Awareness that, in context, some outliers are a valid part of the sample but some outlying data items may be an error in the sample. **Link to**: box and whisker diagrams (SL4.2) and measures of dispersion (SL4.3).
Sampling techniques and their effectiveness.	Simple random, convenience, systematic, quota and stratified sampling methods.

SL 4.2

Content	Guidance, clarification and syllabus links
Presentation of data (discrete and continuous): frequency distributions (tables).	Class intervals will be given as inequalities, without gaps.
Histograms. Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR).	Frequency histograms with equal class intervals. **Not required:** Frequency density histograms.
Production and understanding of box and whisker diagrams.	Use of box and whisker diagrams to compare two distributions, using symmetry, median, interquartile range or range. Outliers should be indicated with a cross. Determining whether the data may be normally distributed by consideration of the symmetry of the box and whiskers.

SL 4.3

Content	Guidance, clarification and syllabus links
Measures of central tendency (mean, median and mode). Estimation of mean from grouped data.	Calculation of mean using formula and technology. Students should use mid-interval values to estimate the mean of grouped data.
Modal class.	For equal class intervals only.
Measures of dispersion (interquartile range, standard deviation and variance).	Calculation of standard deviation and variance of the sample using only technology, however hand calculations may enhance understanding.

	Variance is the square of the standard deviation.
Effect of constant changes on the original data.	**Examples:** If three is subtracted from the data items, then the mean is decreased by three, but the standard deviation is unchanged. If all the data items are doubled, the mean is doubled and the standard deviation is also doubled.
Quartiles of discrete data.	Using technology. Awareness that different methods for finding quartiles exist and therefore the values obtained using technology and by hand may differ.

SL 4.4

Content	Guidance, clarification and syllabus links
Linear correlation of bivariate data. Pearson's product-moment correlation coefficient, r.	Technology should be used to calculate r. However, hand calculations of r may enhance understanding. Critical values of r will be given where appropriate. Students should be aware that Pearson's product moment correlation coefficient (r) is only meaningful for linear relationships.
Scatter diagrams; lines of best fit, by eye, passing through the mean point.	Positive, zero, negative; strong, weak, no correlation. Students should be able to make the distinction between correlation and causation and know that correlation does not imply causation.
Equation of the regression line of y on x. Use of the equation of the regression line for prediction purposes. Interpret the meaning of the parameters, a and b, in a linear regression $y = ax + b$.	Technology should be used to find the equation. Students should be aware: • of the dangers of extrapolation • that they cannot always reliably make a prediction of x from a value of y, when using a y on x line.

SL 4.5

Content	Guidance, clarification and syllabus links
Concepts of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event A is $\mathrm{P}(A) = \frac{n(A)}{n(U)}$. The complementary events A and A' (not A).	Sample spaces can be represented in many ways, for example as a table or a list. Experiments using coins, dice, cards and so on, can enhance understanding of the distinction between experimental (relative frequency) and theoretical probability. Simulations may be used to enhance this topic.
Expected number of occurrences.	**Example:** If there are 128 students in a class and the probability of being absent is 0.1, the expected number of absent students is 12.8.

SL 4.6

Content	Guidance, clarification and syllabus links
Use of Venn diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities.	
Combined events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Mutually exclusive events: $P(A \cap B) = 0$.	The non-exclusivity of "or".
Conditional probability: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.	An alternate form of this is: $P(A \cap B) = P(B)P(A \mid B)$.
	Problems can be solved with the aid of a Venn diagram, tree diagram, sample space diagram or table of outcomes without explicit use of formulae. Probabilities with and without replacement.
Independent events: $P(A \cap B) = P(A)P(B)$.	

SL 4.7

Content	Guidance, clarification and syllabus links
Concept of discrete random variables and their probability distributions. Expected value (mean), $E(X)$ for discrete data. Applications.	Probability distributions will be given in the following ways: X 1 2 3 4 5 $P(X = x)$ 0.1 0.2 0.15 0.05 0.5 $P(X = x) = \frac{1}{18}(4 + x)$ for $x \in \{1, 2, 3\}$; $E(X) = 0$ indicates a fair game where X represents the gain of a player.

SL 4.8

Content	Guidance, clarification and syllabus links
Binomial distribution. Mean and variance of the binomial distribution.	Situations where the binomial distribution is an appropriate model. In examinations, binomial probabilities should be found using available technology. **Not required:** Formal proof of mean and variance. **Link to:** expected number of occurrences (SL4.5).

SL 4.9

Content	Guidance, clarification and syllabus links
The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation.	Awareness of the natural occurrence of the normal distribution. Students should be aware that approximately 68% of the data lies between $\mu \pm \sigma$, 95% lies between $\mu \pm 2\sigma$ and 99.7% of the data lies between $\mu \pm 3\sigma$.
Normal probability calculations.	Probabilities and values of the variable must be found using technology.
Inverse normal calculations	For inverse normal calculations mean and standard deviation will be given. This does not involve transformation to the standardized normal variable z.

SL 4.10

Content	Guidance, clarification and syllabus links
Spearman's rank correlation coefficient, r_s.	In examinations Spearman's rank correlation coefficient, r_s, should be found using technology. If data items are equal, ranks should be averaged.
Awareness of the appropriateness and limitations of Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient, and the effect of outliers on each.	Students should be aware that Pearson's product moment correlation coefficient is useful when testing for only linearity and Spearman's correlation coefficient for any monotonic relationship. Spearman's correlation coefficient is less sensitive to outliers than Pearson's product moment correlation coefficient.

SL 4.11

Content	Guidance, clarification and syllabus links
Formulation of null and alternative hypotheses, H_0 and H_1. Significance levels. p-values.	Students should express H_0 and H_1 as an equation or inequality, or in words as appropriate.
Expected and observed frequencies. The χ^2 test for independence: contingency tables, degrees of freedom, critical value. The χ^2 goodness of fit test.	In examinations: • the maximum number of rows or columns in a contingency table will be 4 • the degrees of freedom will always be greater than one. At SL the degrees of freedom for the goodness of fit test will always be $n-1$ • the χ^2 critical value will be given if appropriate • students will be expected to use technology to find a p-value and the χ^2 statistic • only questions on upper tail tests with commonly-used significance levels (1%, 5%, 10%) will be set • students will be expected to either compare a p-value to the given significance level or compare the χ^2 statistic to a given critical value • expected frequencies will be greater than 5. Hand calculations of the expected values or the χ^2 statistic may enhance understanding. If using χ^2 tests in the IA, students should be aware of the limitations of the test for expected frequencies of 5 or less.
The t-test. Use of the p-value to compare the means of two populations. Using one-tailed and two-tailed tests.	In examinations calculations will be made using technology. At SL, samples will be unpaired, and population variance will always be unknown. Students will be asked to interpret the results of a test. Students should know that the underlying distribution of the variables must be normal for the t-test to be applied. In examinations, students should assume that variance of the two groups is equal and therefore the pooled two-sample t-test should be used.

AHL 4.12

Content	Guidance, clarification and syllabus links
Design of valid data collection methods, such as surveys and questionnaires. Selecting relevant variables from many variables. Choosing relevant and appropriate data to analyse.	Biased and unbiased, personal, unstructured and structured (with consistent answer choices), and precise questioning.
Categorizing numerical data in a χ^2 table and justifying the choice of categorisation. Choosing an appropriate number of degrees of freedom when estimating parameters from data when carrying out the χ^2 goodness of fit test.	Appropriate categories should be chosen with expected frequencies greater than 5.
Definition of reliability and validity. Reliability tests. Validity tests.	Students should understand the difference between reliability and validity and be familiar with the following methods: Reliability: Test-retest, parallel forms. Validity: Content, criterion-related.

AHL 4.13

Content	Guidance, clarification and syllabus links
Non-linear regression.	**Link to:** geometric sequences and series (SL1.3).
Evaluation of least squares regression curves using technology.	In examinations, questions may be asked on linear, quadratic, cubic, exponential, power and sine regression.
Sum of square residuals (SS_{res}) as a measure of fit for a model.	
The coefficient of determination (R^2). Evaluation of R^2 using technology.	R^2 gives the proportion of variability in the second variable accounted for by the chosen model. Awareness that $R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$ and hence = 1 if $SS_{res} = 0$, may enhance understanding but will not be examined. Awareness that many factors affect the validity of a model and the coefficient of determination, by itself, is not a good way to decide between different models. The connection between the coefficient of determination and the Pearson's product moment correlation coefficient for linear models.

AHL 4.14

Content	Guidance, clarification and syllabus links
Linear transformation of a single random variable.	$\mathrm{Var}(X)$ is the expected variance of the random variable X. Variance formula will not be required in examinations. $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$. $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$.
Expected value of linear combinations of n random variables. Variance of linear combinations of n independent random variables.	
$\bar{x}$ as an unbiased estimate of μ.	$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$.
s_{n-1}^2 as an unbiased estimate of σ^2.	$s_{n-1}^2 = \frac{n}{n-1}s_n^2 = \sum_{i=1}^{k} \frac{f_i(x_i - \bar{x})^2}{n-1}$, where $n = \sum_{i=1}^{k} f_i$. Demonstration that $\mathrm{E}(\bar{X}) = \mu$ and $\mathrm{E}(s_{n-1}^2) = \sigma^2$ will not be examined, but may help understanding.

AHL 4.15

Content	Guidance, clarification and syllabus links
A linear combination of n independent normal random variables is normally distributed. In particular, $X \sim \mathrm{N}(\mu, \quad \sigma^2) \Rightarrow \bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$.	
Central limit theorem.	In general, $\bar{X}$ approaches normality for large n, how large depends upon the distribution from which the sample is taken. In examinations, $n > 30$ will be considered sufficient. Online simulations are useful for visualisation.

AHL 4.16

Content	Guidance, clarification and syllabus links
Confidence intervals for the mean of a normal population.	Students should be able to interpret the meaning of their results in context. Use of the normal distribution when σ is known and the t-distribution when σ is unknown, regardless of sample size.

AHL 4.17

Content	Guidance, clarification and syllabus links
Poisson distribution, its mean and variance. Sum of two independent Poisson distributions has a Poisson distribution.	Situations in which it is appropriate to use a Poisson distribution as a model: 1. Events are independent 2. Events occur at a uniform average rate (during the period of interest). Given a context, students should be able to select between the normal, the binomial and the Poisson distributions, recognizing where a particular distribution is appropriate. **Not required:** Formal proof of means and variances for probability distributions.

AHL 4.18

Content	Guidance, clarification and syllabus links
Critical values and critical regions. Test for population mean for normal distribution.	Use of the normal distribution when σ is known and the t-distribution when σ is unknown, regardless of sample size. Samples may be paired or unpaired. The case of matched pairs is to be treated as an example of a single sample technique. Students will not be expected to calculate critical regions for t-tests.
Test for proportion using binomial distribution.	
Test for population mean using Poisson distribution.	Poisson and binomial tests will be one-tailed only.
Use of technology to test the hypothesis that the population product moment correlation coefficient (ρ) is 0 for bivariate normal distributions.	In examinations the data will be given.
Type I and II errors including calculations of their probabilities.	Applied to normal with known variance, Poisson and binomial distributions. For discrete random variables, hypothesis tests and critical regions will only be required for one-tailed tests. The critical region will maximize the probability of a Type I error while keeping it less than the stated significance level.

AHL 4.19

Content	Guidance, clarification and syllabus links
Transition matrices. Powers of transition matrices.	In general, the column state matrix (s_n) after n transitions is given by $s_n = T^n s_0$, where T is the transition matrix, with T_{ij} representing the probability of moving from state j to state i, and s_0 is the initial state matrix. Use of transition diagrams to represent transitions in discrete dynamical systems.
Regular Markov chains. Initial state probability matrices.	
Calculation of steady state and long-term probabilities by repeated multiplication of the transition matrix or by solving a system of linear equations.	Examination questions will state when exact solutions obtained from solving equations are required. Awareness that the solution is the eigenvector corresponding to the eigenvalue equal to 1. **Link to:** matrices (AHL1.14), eigenvalues (AHL1.15) and adjacency matrices (AHL3.15).

SL 5.1

Content	Guidance, clarification and syllabus links
Introduction to the concept of a limit.	Estimation of the value of a limit from a table or graph. **Not required:** Formal analytic methods of calculating limits.
Derivative interpreted as gradient function and as rate of change.	Forms of notation: $\frac{\mathrm{d}y}{\mathrm{d}x}$, $f'(x)$, $\frac{\mathrm{d}V}{\mathrm{d}r}$ or $\frac{\mathrm{d}s}{\mathrm{d}t}$ for the first derivative. Informal understanding of the gradient of a curve as a limit.

SL 5.2

Content	Guidance, clarification and syllabus links
Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0, f'(x) = 0, f'(x) < 0$.	Identifying intervals on which functions are increasing ($f'(x) > 0$) or decreasing ($f'(x) < 0$).

SL 5.3

Content	Guidance, clarification and syllabus links
Derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}, n \in \mathbb{Z}$ The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$ where all exponents are integers.	

SL 5.4

Content	Guidance, clarification and syllabus links
Tangents and normals at a given point, and their equations.	Use of both analytic approaches and technology.

SL 5.5

Content	Guidance, clarification and syllabus links
Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$, where $n \in \mathbb{Z}, \quad n \neq -1$.	Students should be aware of the link between anti-derivatives, definite integrals and area.
Anti-differentiation with a boundary condition to determine the constant term.	**Example:** If $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + x$ and $y = 10$ when $x = 1$, then $y = x^3 + \frac{1}{2}x^2 + 8.5$.
Definite integrals using technology. Area of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x) > 0$.	Students are expected to first write a correct expression before calculating the area, for example $\int_2^6 (3x^2 + 4)\mathrm{d}x$. The use of dynamic geometry or graphing software is encouraged in the development of this concept.

SL 5.6

Content	Guidance, clarification and syllabus links
Values of x where the gradient of a curve is zero. Solution of $f'(x) = 0$. Local maximum and minimum points.	Students should be able to use technology to generate $f'(x)$ given $f(x)$, and find the solutions of $f'(x) = 0$. Awareness that the local maximum/minimum will not necessarily be the greatest/least value of the function in the given domain.

SL 5.7

Content	Guidance, clarification and syllabus links
Optimisation problems in context.	**Examples:** Maximizing profit, minimizing cost, maximizing volume for a given surface area. In SL examinations, questions on kinematics will not be set.

SL 5.8

Content	Guidance, clarification and syllabus links
Approximating areas using the trapezoidal rule.	Given a table of data or a function, make an estimate for the value of an area using the trapezoidal rule, with intervals of equal width. **Link to:** upper and lower bounds (SL1.6) and areas under curves (SL5.5).

AHL 5.9

Content	Guidance, clarification and syllabus links
The derivatives of sin x, cos x, tan x, e^x, ln x, x^n where $n \in \mathbb{Q}$. The chain rule, product rule and quotient rules. Related rates of change.	**Link to**: maximum and minimum points (SL5.6) and optimisation (SL5.7).

AHL 5.10

Content	Guidance, clarification and syllabus links
The second derivative.	Both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$ for the second derivative.
Use of second derivative test to distinguish between a maximum and a minimum point.	Awareness that a point of inflexion is a point at which the concavity changes and interpretation of this in context.
	Use of the terms "concave-up" for $f''(x) > 0$, and "concave-down" for $f''(x) < 0$. **Link to:** kinematics (AHL5.13) and second order differential equations (AHL5.18).

AHL 5.11

Content	Guidance, clarification and syllabus links
Definite and indefinite integration of x^n where $n \in \mathbb{Q}$, including $n = -1$, $\sin x$, $\cos x$, $\frac{1}{\cos^2 x}$ and e^x.	
Integration by inspection, or substitution of the form $\int f(g(x))g'(x)\mathrm{d}x$.	**Examples:** $\int \sin(2x+5)\mathrm{d}x$, $\int \frac{1}{3x+2}\mathrm{d}x$, $\int 4x\sin x^2\,\mathrm{d}x$, $\int \frac{\sin x}{\cos x}\,\mathrm{d}x$.

AHL 5.12

Content	Guidance, clarification and syllabus links
Area of the region enclosed by a curve and the x or y-axes in a given interval.	Including negative integrals.
Volumes of revolution about the x- axis or y- axis.	$V = \int_a^b \pi y^2\mathrm{d}x$ or $V = \int_a^b \pi x^2\mathrm{d}y$

AHL 5.13

Content	Guidance, clarification and syllabus links
Kinematic problems involving displacement s, velocity v and acceleration a.	$v = \frac{\mathrm{d}s}{\mathrm{d}t}$; $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = v\frac{\mathrm{d}v}{\mathrm{d}s}$. Displacement = $\int_{t_1}^{t_2} v(t)\mathrm{d}t$. Total distance travelled = $\int_{t_1}^{t_2} \lvert v(t)\rvert\mathrm{d}t$. Speed is the magnitude of velocity.
	Use of $\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t}$ and $\ddot{x} = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$.

AHL 5.14

Content	Guidance, clarification and syllabus links
Setting up a model/differential equation from a context.	**Example:** The growth of an algae G, at time t, is proportional to $\sqrt{G}$.
Solving by separation of variables.	**Example:** An exponential model as a solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = ky$. The term "general solution".

AHL 5.15

Content	Guidance, clarification and syllabus links
Slope fields and their diagrams.	Students will be required to use and interpret slope fields.

AHL 5.16

Content	Guidance, clarification and syllabus links
Euler's method for finding the approximate solution to first order differential equations. Numerical solution of $\frac{dy}{dx} = f(x, y)$.	Spreadsheets should be used to find approximate solutions to differential equations. In examinations, values will be generated using permissible technology.
Numerical solution of the coupled system $\frac{dx}{dt} = f_1(x, y, t)$ and $\frac{dy}{dt} = f_2(x, y, t)$.	Contexts could include predator-prey models.

AHL 5.17

Content	Guidance, clarification and syllabus links
Phase portrait for the solutions of coupled differential equations of the form: $\frac{dx}{dt} = ax + by$ $\frac{dy}{dt} = cx + dy$. Qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues. Sketching trajectories and using phase portraits to identify key features such as equilibrium points, stable populations and saddle points.	Systems will have distinct, non-zero, eigenvalues. If the eigenvalues are: • Positive or complex with positive real part, all solutions move away from the origin • Negative or complex with negative real part, all solutions move towards the origin • Complex, the solutions form a spiral • Imaginary, the solutions form a circle or ellipse • Real with different signs (one positive, one negative) the origin is a saddle point. Calculation of exact solutions is only required for the case of real distinct eigenvalues. **Link to:** eigenvectors and eigenvalues (AHL1.15).

AHL 5.18

Content	Guidance, clarification and syllabus links
Solutions of $\frac{d^2x}{dt^2} = f(x, \frac{dx}{dt}, t)$ by Euler's method.	Write as coupled first order equations $\frac{dx}{dt} = y$ and $\frac{dy}{dt} = f(x, y, t)$. Solutions of $\frac{d^2x}{dt^2} + a\frac{dx}{dt} + b = 0$, can also be investigated using the phase portrait method in AHL 5.17 above. Understanding the occurrence of simple second order differential equations in physical phenomena would aid understanding but in examinations the equation will be given.